PH5840 Lecture 11

Note Title 9/3/2015

Bell-CHSH inequality

(1) Correlations in the EPR/Bell states :-

$$|\beta_{11}\rangle_{AB} = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Spin-singlet state.

(combined state of two spin-$\frac{1}{2}$ particles)

* Measuring σ_z component of Spin-A

$$p(-1) = \mathrm{Tr}\left[\left(|1\rangle\langle 1|_A \otimes \mathbb{I}_B\right)\left(|\beta_{11}\rangle\langle\beta_{11}|\right)\right]$$

$$= \mathrm{Tr}\left(+\frac{1}{2}\left(|10\rangle\langle 10| - \frac{1}{2}|10\rangle\langle 01|\right)\right)$$

$$= \underline{\underline{\frac{1}{2}}} \;. = p(+1)$$

* If the outcome is +1 for spin-A, we know that the outcome is -1 for spin-B. (Vice-versa)

* Same anti-correlations hold, no matter which spin-direction we choose to measure in.

Eg. Measure along $\vec{v}\cdot\vec{\sigma}$:- Let $\{|v_\pm\rangle\}$ be eigenstates of $\vec{v}\cdot\vec{\sigma}$.

$$|0\rangle = \alpha|v_+\rangle + \beta|v_-\rangle$$
$$|1\rangle = \gamma|v_+\rangle + \delta|v_-\rangle$$

$$U = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}$$

$$|U| = e^{i\theta}$$

$$\therefore \quad \frac{|01\rangle - |10\rangle}{\sqrt{2}} = \underbrace{(\alpha\delta - \beta\gamma)}_{\text{overall phase!}} \underbrace{\left(\frac{|v_+ v_-\rangle - |v_- v_+\rangle}{\sqrt{2}} \right)}_{\text{anti-correlated state.}}$$

(2) An experimental test of quantum correlations?

<u>Bell - CHSH inequality:-</u>

Binary valued observables

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$P \in \{\pm 1\}$	$\longleftrightarrow$ joint state of AB	$R \in \{\pm 1\}$
$Q \in \{\pm 1\}$		$S \in \{\pm 1\}$

<u>Assumptions:-</u>

(i) P, Q, R, S are objective properties of the system:-
Act of measurement merely "reveals" their values.

(ii) A & B perform their measurements at the <u>same time</u>
(causally disconnected)

$\therefore$ It is possible to assign joint values to P, Q, R, S simultaneously.

- Consider the observable M:

$$M = PR + QR + QS - PS = (P+Q)R + (Q-P)S.$$

Since $P, Q \in \{\pm 1\}$, $M = \pm 2$

- Let $f(p, q, r, s)$: joint prob. distbn. over the outcomes

Then, $\mathbb{E}(PQ + QR + QS - PS) = \sum_{p,q,r,s} f(\;)(pr + qr + qs - ps)$

$$\leq 2$$

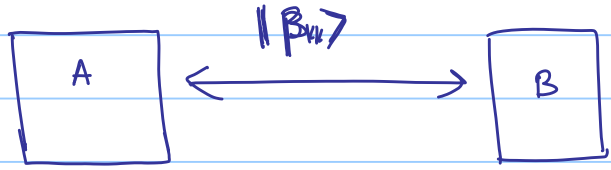
Also, $\mathbb{E}(PR + QR + QS - PS) = \mathbb{E}(PR) + \mathbb{E}(QR) + \mathbb{E}(QS) - \mathbb{E}(PS)$

$$\therefore \quad \boxed{\mathbb{E}(PR) + \mathbb{E}(QR) + \mathbb{E}(QS) - \mathbb{E}(PS) \leq 2}$$

- CHSH inequality: Clauser, Horne, Shimony & Holt (1969)
- One of the set of Bell inequalities (1964)
- Can be experimentally verified!

• For quantum states and measurements:-

Suppose joint state is $|\psi\rangle = \dfrac{|01\rangle - |10\rangle}{\sqrt{2}} = |\beta_{11}\rangle$



$P_A = \sigma_z$

$Q_A = \sigma_x$

$R_B = \dfrac{-\sigma_z - \sigma_x}{\sqrt{2}}$

$S_B = \dfrac{\sigma_z - \sigma_x}{\sqrt{2}}$

$$\mathbb{E}(PR) \equiv \langle P_A \otimes R_B \rangle_{|\beta_{11}\rangle}$$

$$= \langle \beta_{11} | \sigma_z \otimes \frac{-\sigma_z - \sigma_x}{\sqrt{2}} | \beta_{11} \rangle$$

$$= -\left(\frac{\langle 01| - \langle 10|}{\sqrt{2}}\right)\left(\sigma_z \otimes \frac{(\sigma_z + \sigma_x)}{\sqrt{2}}\right)\left(\frac{|01\rangle - |10\rangle}{\sqrt{2}}\right)$$

$$= \left(\frac{\langle 10| - \langle 01|}{2\sqrt{2}}\right)\left[-|01\rangle + |00\rangle + |10\rangle + |11\rangle\right]$$

$$= \frac{1+1}{2\sqrt{2}} = \frac{1}{\sqrt{2}} \;//$$

Illy $\mathbb{E}(QR) = \langle \beta_{11} | \sigma_x \otimes \left(\dfrac{-\sigma_z - \sigma_x}{\sqrt{2}}\right) | \beta_{11} \rangle = \dfrac{1}{\sqrt{2}}$

$$\mathbb{E}(QS) = \langle \beta_{11} | \sigma_x \otimes \left(\frac{\sigma_z - \sigma_x}{\sqrt{2}} \right) | \beta_{11} \rangle = \frac{1}{\sqrt{2}}$$

$$\mathbb{E}(PS) = \langle \beta_{11} | \sigma_z \otimes \left(\frac{\sigma_z - \sigma_x}{\sqrt{2}} \right) | \beta_{11} \rangle = \frac{-1}{\sqrt{2}}$$

$$\therefore \; \mathbb{E}(PR) + \mathbb{E}(QR) + \mathbb{E}(QS) - \mathbb{E}(PS) = \underline{\underline{2\sqrt{2}}} \; !!$$

* violation of the Bell-CHSH inequality observed using photons by Alain Aspect et al. (1982)

* Question our "reasonable" assumptions:
(i) objective reality of physical properties (Realism)
(ii) Locality
(iii) Existence of joint prob-distribution : joint-measurability

→ Recent expt. using e^- spins: arxiv.org/abs/1508.05949